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Tool path generation method for multi-axis machining of helical milling cutter with specific cross-section profile

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Abstract

The geometry of helical milling cutter with specific cross-section profile is normally generated with form cutter by the toolpath generating method. The general purpose CAD/CAM software can generate the cutter location file of this class of product only by the sculpturing method. To obtain the flexibility of machining process, this paper presents an interference-free toolpath generating method for semi-finish and finish machining of the helical milling cutter. The method for design and machining of the helical milling cutter is established based on the differential geometry and the enveloping theory. In the finish machining of helical milling cutter, the form grinding wheel profile is firstly derived. To reduce the cost and lead time of the generating method with the form-mill cutter for semi-finish machining, a toolpath generating method which combines the advantages of the generating method and sculpturing method was presented. The cutter location uses ball-nose conical end mill is derived according to the geometric characteristics of the enveloping element. The cutting simulations with solid model were performed to verify the proposed toolpath generation method.

Keywords: Enveloping theory; Multi-axis machining; Toolpath; Helical milling cutter

1. Introduction

The geometry of helical milling cutter with specific cross-section profile is normally generated with form grinding wheel by special multi-axis machine and software. In this so-called "generating method", the cutting tool follows the same path as the enveloping element relative to the helical milling cutter. The enveloping element is derived from the reverse enveloping theory [1]. Using the generating method, the tool geometry and dimension must be same as that of the enveloping element. However, the wear of the tool is inevitable. Since the worn tool could not be reused after dressing, a number of cutting tools need to be prepared. With the increasing use of computers in design and manufacturing, the processes for generating the workpiece surface can be obtained by general purpose CAD/CAM software. However, the commercial CAD/CAM software can generate the cutter location file of helical milling cutter only by the sculpturing method. To obtain the flexibility of machining process, this paper presents an interference-free toolpath generating method for semi-finish and finish machining of helical milling cutter.

In the design and manufacture for enveloping surfaces, Litvin and Gutman [2] investigated machine tool setting for different manufacturing methods of hypoid and spiral bevel gears. Ivanov et al. [3] proposed a mathematical model for the determination of the profile of the helical surface when the tool geometry used and the parameters (machine setup) determining the tool orientation towards the blank were specified. Yan and Liu [4] and Lee and co-workers [5] integrated the activities for design and

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manufacturing of the variable pitch lead screw and roller gear cam.

In the application of reverse enveloping theory, Kang et al. [6] published a general mathematical approach for the resolution of both inverse and direct problems with disk and axial-type tools for the manufacture of helical grooves. The approach is based on the establishment of the fundamental analytical conditions of engagement between the generating tool surface and generated helical groove surface. Ivanov et al. [7] presented a generalized analytical method for profiling all types of rotation tools for forming helical surfaces. Research was carried out for the influence of the input information of profiling and adjustment of the tool on the accuracy of the helical surface. A specialized CAD system was developed for the practical application.

This paper describes the toolpath generation method for semi-finish machining and finish machining of helical milling cutter. To demonstrate the proposed toolpath generation, a plain milling cutter with helix angle is selected. In the multi-axis machining of plain milling cutter, the form grinding wheel profile is first derived. The cutter location of the generating method is represented. Owing to the limited choice of cutting tool used by the generating method, this paper presents an interference-free toolpath generating method for helical groove machining. According to the Euler angle representations and homogeneous coordinate transformation, the cutter location and orientation using a ball-nose conical end mill are generated.

2. Surface generation of plain milling cutter

2.1 Equation of cross-section profile

The design parameters of a plain milling cutter with helix angle are specified firstly for the determination of the cross-section profile. As shown in Fig. 1, D_1 and D_2 are the outer and the root diameters; τ_z is the nominal helix surface angle; r_c is the fillet radius; l is the land width; α , β , γ , and δ are the radial rake angle, the clearance angle, the relief angle and the index angle, respectively. The centre point $Q(x_q, y_q)$, the tangent points $B(x_b, y_b)$ and $D(x_d, y_d)$, and point $E(x_e, y_e)$ are calculated as follows:

Centre point $Q(x_q, y_q)$:

$$x_q = y_q \tan\alpha + r_c C\alpha - (0.5D_1 - r_c S\alpha)\tan\alpha ;$$

$$y_{q1}, y_{q2} = \frac{-B_q \pm \sqrt{B_q^{\,2} - 4A_q C_q}}{2A_q} \tag{1}$$

$$y_q = \max(y_{q1}, y_{q2})$$

where

$$A_q = 1 + \tan^2\alpha$$

$$B_q = 2C_1 \tan\alpha$$

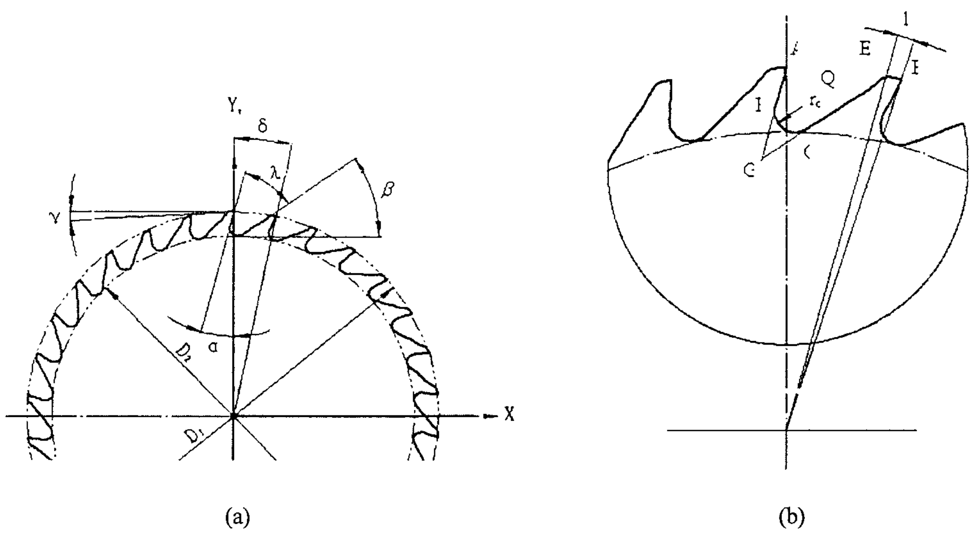


Fig. 1. The cross-section profile of a plain milling cutter with helix angle.

$$C_q = C_1^2 - (0.5D_2 + r_c)^2$$
$$C_1 = r_c C\alpha - (0.5D_1 - r_c S\alpha)\tan\alpha$$
$$(x_q, y_q) = (e, f)$$

Tangent points $B(x_b, y_b)$ and $D(x_d, y_d)$:

$$x_b = (y_b - 0.5D_1)\tan\alpha \; ; \; y_b = \frac{-B_b \pm \sqrt{B_b^2 - 4A_bC_b}}{2A_b} \tag{2}$$

where

$$A_b = 1 + \tan^2\alpha$$
$$B_b = -D_1 \tan^2\alpha - 2e\tan\alpha - 2f$$
$$C_b = (0.5D_1)^2 \tan^2\alpha + eD_1\tan\alpha + e^2 + f^2 - r_c^2$$

and

$$y_d = x_d + C_2 \; ; \; x_d = \frac{-B_d \pm \sqrt{B_d^2 - 4A_dC_d}}{2A_d} \tag{3}$$

where

$$A_d = 1 + \tan^2\beta$$
$$B_d = 2C_2\tan\beta - 2f\tan\beta - 2e$$
$$C_d = C_2^2 - 2C_2 f + e^2 + f^2 - r_c^2$$
$$C_2 = 0.5D_1(C\delta - S\delta\tan\beta) + lS(\delta-\gamma) + lC(\delta-\gamma)\tan\beta$$

The transition point B (or D) is tangential if $B_b^2 - 4A_bC_b = 0$ (or $B_d^2 - 4A_dC_d = 0$).

Point $E(x_e, y_e)$:

$$x_e = 0.5D_1S\delta - lC(\delta-\gamma) \; ; \; y_e = 0.5D_1C\delta + lS(\delta-\gamma) \tag{4}$$

2.2 Surface equation of plain milling cutter

The curve set of the cross-section profile represented in the coordinate system $(OXYZ)_p$ can be expressed as follows:

$$R_t = [x_t \quad y_t \quad 1]^T \tag{5}$$

The helicoid is generated by the cross-section profile performing a screw motion. The generated surface and the normal vector of the helicoid are derived as follows:

$$R_P = [x_t C\tau - y_t S\tau \quad x_t S\tau + y_t C\tau \quad p\tau \quad 1]^T \tag{6}$$

$$N_p = \frac{\partial R_p}{\partial t} \times \frac{\partial R_p}{\partial \tau} = N_p(t, \tau) \tag{7}$$

where τ and p are the angle of rotation and the screw parameter in the screw motion, respectively. "C" and "S" refer to the cosine and sine functions, respectively.

3. Toolpath generation methods

3.1 Generating method

The generating method employs the form cutter according to the geometry of enveloping element, and the kinematics relations of cutting tool with respect to the workpiece. Machining by the generating method has some advantages such as the machining pass of the generating method is fewer than that of sculptured method and it is capable of creating very smooth surface finishes. Because of these characteristics, the generating method is generally performed in the industry [5].

3.1.1 Cutter location derivation of disk-shaped tool

The surface of helical milling cutter can be identified as the envelope to the family of surfaces of form cutter. The family of surfaces is generated in relative screw motion that is performed with a constant screw parameter. Fig. 2 and Fig. 3 show a form cutter used in multi-axis machining for helical surface. Coordinate systems $(OXYZ)_f$, $(OXYZ)_p$, $(OXYZ)_c$ and $(OXYZ)_{Tip}$ that are rigidly connected to the frame of the cutting machine, the helical milling cutter, the tool, and the tip of the cutting tool, respectively. Axis Z_P is the axis of the helical milling cutter. While the blank rotates through angle φ, the tool translates along the Z_f axis with the distance $p\varphi$. r_P is the pitch radius of the helical milling cutter; r_c is the cutter mean radius; A_c and ψ are the shortest distance and the crossing angle between the axes of rotation of the tool and the helical milling cutter.

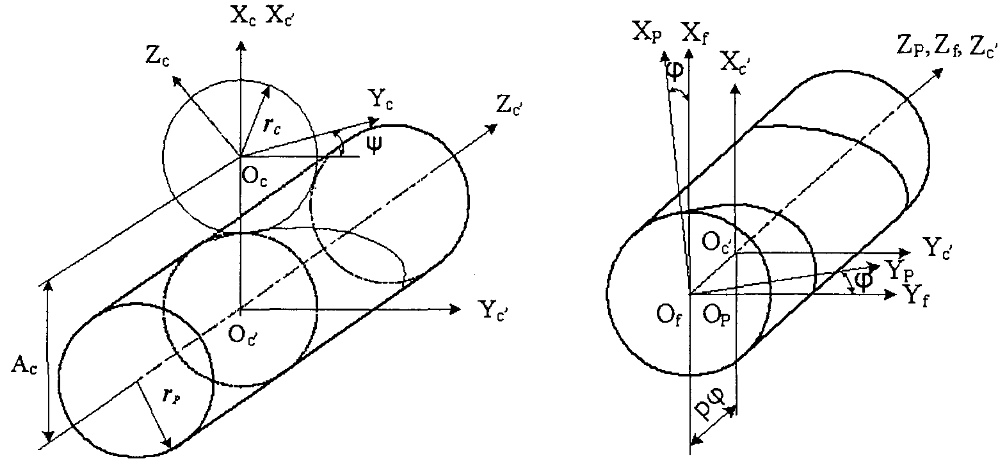


Fig. 2. Coordinate systems applied for the helical milling cutter by disk-shaped tool.

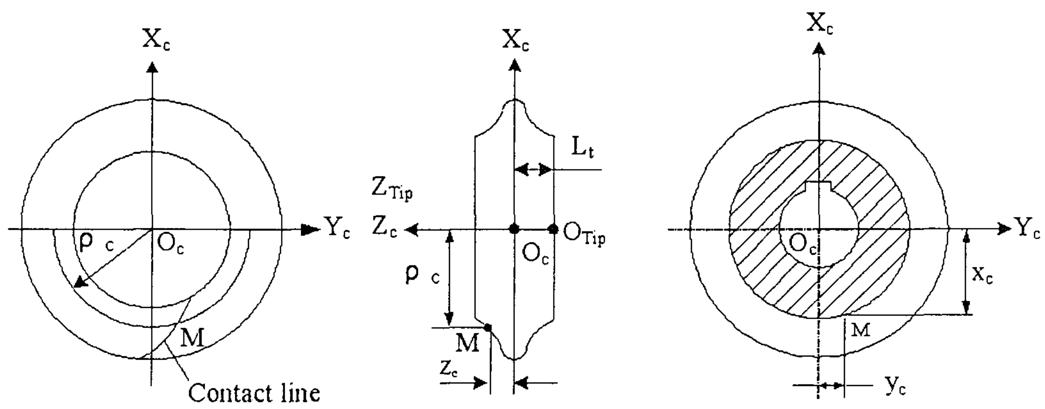


Fig. 3. Schematic illustration of the cross-section profiles of form cutter.

By applying the homogeneous coordinate transformation matrix ${}^{P}A_{Tip} = {}^{P}A_{f}\,{}^{f}A_{c'}\,{}^{c'}A_{c}\,{}^{c}A_{Tip}$, the cutter location of the coordinate systems $(OXYZ)_{Tip}$ with respect to $(OXYZ)_{P}$ is as follows:

$$
{}^{P}A_{Tip} = \begin{bmatrix} C\varphi & -C\psi S\varphi & S\psi S\varphi & -L_t S\psi S\varphi + A_c C\varphi \\ S\varphi & C\psi C\varphi & -S\psi C\varphi & L_t S\psi C\varphi + A_c S\varphi \\ 0 & S\psi & C\psi & L_t C\psi + p\varphi \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}
$$

where the third column is the components of tool orientation, and the fourth column is the components of tool location. The suffix *Tip* denotes the coordinate system at the tool tip.

3.1.2 Cross-section profile of disk-shaped tool

The determination of the tool surface that generates the helical milling cutter is based on the reverse enveloping theory. This method describes the condition that the line of tangency between workpiece surface and tool surface is such a one at which the normals to workpiece surface intersect the rotation axis of the disk-shaped tool [1].

The common normal to workpiece surface and tool surface is represented by the following equation.

$$\frac{X_P - x_P(t,\tau)}{N_{x_P}(t,\tau)} = \frac{Y_P - y_P(t,\tau)}{N_{y_P}(t,\tau)} = \frac{Z_P - z_P(t,\tau)}{N_{z_P}(t,\tau)} \tag{9}$$

Here (X_P, Y_P, Z_P) are the coordinates of the point of intersection of the normal with the Z_c-axis of the tool in $(OXYZ)_P$.

Considering the position when $(OXYZ)_P$ coincides with $(OXYZ)_f$ ($\varphi = 0$). The equation of meshing is

$$\begin{aligned} f(t,\tau) = -n_{xP}(y_P C\psi + z_P S\psi) + \\ (n_{yP} C\psi + n_{zP} S\psi)(x_P - A_c) = 0 \end{aligned} \tag{10}$$

There is a family of contact lines on the workpiece surface. One of the single contact lines on the workpiece surface can be determined with $\varphi = 0$. This line is obtained by the equations,

$$R_P = R_P(t,\tau), \quad f(t,\tau) = 0$$

The contact line on the cutter surface is determined by Eq. (11),

$$R_c(t,\tau) = {}^cA_f\,{}^fA_P R_P(t,\tau), \quad f(t,\tau) = 0 \tag{11}$$

Fig. 3 shows the line of tangency on the cutter surface; M is the current point of this line with coordinates (x_c, y_c, z_c). The profile of the tool obtained by intersection of the tool surface by plane $y_c = 0$ can be represented by coordinates (x_c, z_c). By using the equation of meshing and considering τ as the input data, the respective value t is obtained. Knowing the couple (t,τ), the coordinates (x_c, y_c, z_c) of the contact line from Eq. (11) is determined. The tool axial profile can be obtained as follows:

$$\rho_c = (x_c^2 + y_c^2)^{0.5} = \rho_c(t,\tau) \tag{12}$$

$$x_c(\tau) = -\rho_c(\tau), z_c(\tau) \tag{13}$$

3.2 Combined generating and sculpturing method

To reduce the cost and lead time of the generating method with the form cutter for semi-finish machining, the combined generating and sculpturing method with a ball-nose conical end mill for machining of helical groove is proposed here. The cutter location is derived according to the geometric characteristics of the enveloping element. The machining path is along the parametric curve of plain milling cutter.

3.2.1 Derivation of the cutter location

To express the position and the orientation of the tool, a coordinate system is defined at the point of tangency of mating surfaces. As shown in Fig. 4, the coordinate system is denoted as $(OXYZ)_L$. The X_L axis is defined as the tangent vector at the joint point of the axial profile of the enveloping element. The Y_L axis is in the surface normal direction. The Z_L axis forms a right-hand triad with X_L and Y_L axes.

Cutter orientation is defined in the coordinate system $(OXYZ)_L$ by Euler angle representations $R(\varphi_t, \eta_t, \phi_t)$ [8]. The coordinate system $(OXYZ)_L$ is first rotated with a rotation angle φ_t about Y_L axis to form the coordinate system $(OXYZ)_{L'}$; it is then rotated with an inclination angle η_t about $Z_{L'}$ axis to form the coordinate system $(OXYZ)_{L''}$. The $X_{L''}$ axis is specified to be the orientation of cutter. To locate a cutter tangent to the enveloping element at the contact point of the mating surfaces, φ_t is set to be 0^0 and η_t is set to be $-b_t$. The cutter location with respect to the coordinate system $(OXYZ)_P$ of plain milling cutter can be obtained by homogeneous coordinate transformation.

$${}^PA_{Tip} = {}^PA_L\,{}^LA_{Tip} = \begin{bmatrix} I_x & J_x & K_x & Q_x \\ I_y & J_y & K_y & Q_y \\ I_z & J_z & K_z & Q_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{14}$$

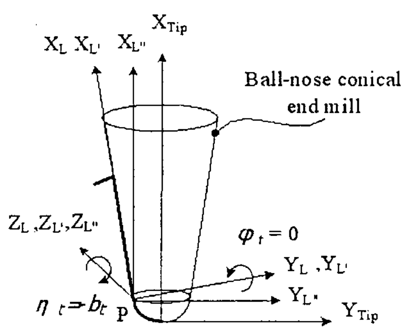
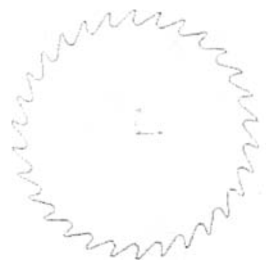
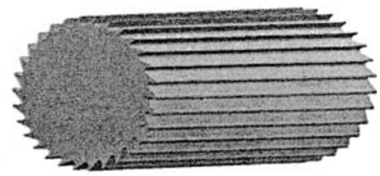


Fig. 4. Coordinate systems of the ball-nose conical end mill.

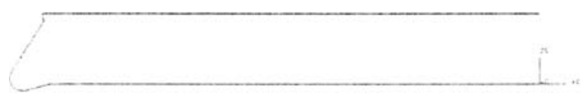


(a) The cross-section profile.

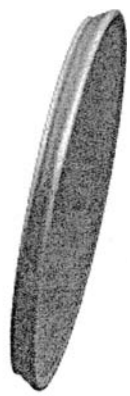


(b) The surface geometry.

Fig. 5. Surface geometry of plain milling cutter with helix angle.



(a) The cross-section profiles.



(b) Surface of revolution.

Fig. 6. Surface geometry of form grinding wheel.

where I_x, I_y, I_z are the components of tool orientation, and Q_x, Q_y, Q_z are the components of tool location.

4. Implementation

4.1 Design parameter and surface geometry of plain milling cutter

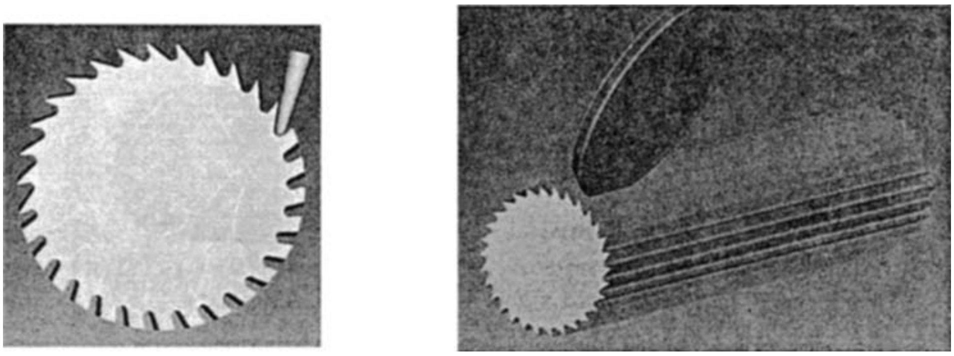
A designed plain milling cutter with the following parameters: $D_1 = 163.5\,mm$, $D_2 = 144\,mm$, $l = 2\,mm$, $\tau_z = 2^\circ$, $r_c = 3\,mm$, $\alpha = 15^\circ$, $\delta = 12^\circ$, $\lambda = 45^\circ$, is used to verify the validity and effectiveness of the proposed methods. By substituting these parameters into Eqs. (1)-(6), the surface geometry of the plain milling cutter is obtained as shown in Fig. 5.

4.2 Surface geometry of form grinding wheel

Based on the reverse enveloping theory, the enveloping element (disk-shaped tool) which mates with the plain milling cutter is derived. In this paper, the shortest distance A_c and the crossing angle ψ between the axes of rotation of the tool and the plain milling cutter are set to be $200\,mm$ and 90°. Fig. 6(a) shows the cross-section profile of form grinding wheel. Fig. 6(b) shows the revolution surface of the cross-section profile.

4.3 Toolpath simulation

To avoid collision between all machine tool components, the generated toolpath is verified before actual machining through solid cutting simulation. Fig. 7(a) presents the process of semi-finish machining by the proposed method with ball-nose conical end mill simulated by VERICUT® software. Fig. 7(b) shows the process of finish machining. The simulation results demonstrate that the machining process works well and the collision between the shank and workpiece surface will not occur.



(a) Semi-finish machining (b) Finish machining.

Fig. 7. Simulating of plain milling cutter cutting by the proposed methods.

5. Conclusion

In order to obtain the flexibility of the machining, this paper presents the interference-free toolpath generation method for multi-axis machining of helical milling cutter. The geometry of the form grinding wheel is derived in accordance with the reverse enveloping theory. Through the homogeneous coordinate transformation, the cutter location for finish machining using form grinding wheel is generated. The toolpath of the semi-finish machining with collision-free is proposed. The ball-nose conical end mill is confined within the cross-section profile of form cutter. The cutting simulations with solid model are performed to verify the proposed toolpath generation methods. The proposed methodology can be used to automate the programming of toolpaths for the machining of helical milling cutter.

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References

[1] F. L. Litvin, Gear Geometry and Applied Theory, *PTR* Prentice Hall, New Jersey (1994).

[2] F. L. Litvin and Y. Gutman, Methods of synthesis and analysis for hypoid gear-drives of "Formate" and "Helixform", *ASME Trans., J. Mech. Des.* 103 (1981) 83-88.

[3] V. Ivanov, G. Nankov and V. Kirov, CAD orientated mathematical model for determination of profile helical surfaces, *Int. J. Mach. Tools Manufact.* 38 (1998) 1001-1015.

[4] H. S. Yan and J. Y. Liu, Geometry design and machining of variable pitch lead screws with cylindrical meshing elements, *ASME Trans., J. Engng Ind.* 115 (1993) 490-495.

[5] R. S. Lee and J. N. Lee, A new method of tool orientation determination by enveloping element for 5-axis machining of spatial cam, *Int. J. Prod. Res.* 40 (2002) 2379-2398.

[6] S. K. Kang, K. F. Ehmann and C. Lin, A CAD approach to helical groove machining- I. Mathematical model and model solution, *Int. J. Mach. Tools Manufact.* 36 (1996) 141-153.

[7] V. Ivanov and G. Nankov, Profiling of rotation tools for forming of helical surfaces, *Int. J. Mach. Tools Manufact.* 38 (1998) 1125-1148.

[8] R. P. Paul, Robot Manipulators: Mathematics, Programming and Control, MIT Press, Cambridge, Mass (1981).